M-math 2nd year Back paper Exam
Subject : Fourier Analysis

Time : 3.00 hours Max.Marks 60.

Answer any two questions from each part. Contact : rajeevbee@hotmail.com

**Part A**

1. Suppose that the approximate identity $K(r,\theta), r \to 0$ satisfies $K(r,\theta) = K(r,-\theta), \theta \in (-\pi,\pi]$. Suppose that $\phi \in L^\infty((-\pi,\pi])$ satisfies $\lim_{t\to 0}\{\phi(t)+\phi(-t)\} = 2L$. Show that

$$\lim_{r\to 0} \frac{1}{2\pi}\int_{-\pi}^{\pi} K(r,\theta)\phi(\theta)\ d\theta = L.$$

(15)

2. Prove that if the Fourier partial sums of $f \in L^p, 1 \le p < \infty$ converge in $L^p$, then the Fejér means converge in $L^p$. (15)

3. Let $f \in L^2(-\pi,\pi]$ be absolutely continuous i.e. $f(t) = \int_{-\pi}^{t} f'(s)ds$ with $f' \in L^2$. Use Parseval's identity to prove that $\|f'\|_2^2 = \sum_{-\infty}^{\infty} n^2|\hat{f}(n)|^2$. (15)

**Part B**

4. a) Use the Fourier reciprocity relation with an appropriate choice of kernel and a suitable integrable function to show that

$$\lim_{t\to 0}\int_{-\infty}^{\infty} \frac{sin2\pi\xi}{\pi\xi} e^{-4\pi^2 t\xi^2} d\xi = 1.$$

b) Let $f(x) = I_{(a,b)}(x)$. Compute the limit $\int_{-M}^{M} \hat{f}(\xi)e^{2\pi i x\xi}d\xi$ as $M \to \infty$. (9+6.)

5. Let $f(x) := I_{[-1,1]}(x), x \in \mathbb{R}$. Let $H(t,x)$ be the heat kernel and for $h : \mathbb{R} \to \mathbb{R}$, let $\|h\|_\infty := ess.sup.|h(x)|$. Show that $\|H_t f - f\|_\infty$ does not go to zero as $t \to 0$. (15)

6. Suppose $K(x) \in L^1(\mathbb{R}^n)$ and $K_t(x) := t^{-n}K(x/t), t > 0$. If $K \ge 0$ and $\int K(x)dx = 1$, show that $K_t(x), t \to 0$ is an approximate identity. (15)